

Section 8.3 Solutions

Section 8.3: Systems of Linear Equations – Determinants

#1 – 8: Find the value of the determinant of the following matrices

1) $\begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$

$$4 \times 2 - 5 \times 1$$
$$8 - 5$$

ANSWER 3

3) $\begin{bmatrix} 3 & -5 \\ 0 & -2 \end{bmatrix}$

$$3 \times -2 - 0 \times -5$$
$$-6 - 0$$

ANSWER -6

$$5) \begin{bmatrix} 3 & 5 & -1 \\ 4 & 2 & 3 \\ 2 & 5 & 6 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 4 & 2 \\ 2 & 5 \end{bmatrix}$$

$$3 \cdot 2 \cdot 6 + 5 \cdot 3 \cdot 2 + (-1) \cdot 4 \cdot 5 = 46$$

$$36 + 30 - 20 = 46$$

$$2 \cdot 2 \cdot (-1) + 5 \cdot 3 \cdot 3 + 6 \cdot 4 \cdot 5 = 161$$

$$-4 + 45 + 120 = 161$$

$$\text{Determinant} = 46 - 161$$

ANSWER -115

$$7) \begin{bmatrix} -3 & 2 & 8 \\ 0 & 6 & -3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 0 & 6 \\ 2 & 1 \end{bmatrix}$$

$$-3 \cdot 6 \cdot 4 + 2 \cdot 3 \cdot 2 + 8 \cdot 0 \cdot 1 = -84$$

$$-72 - 12 + 0 = -84$$

$$2 \cdot 6 \cdot 8 + 1 \cdot (-3) \cdot (-3) + 4 \cdot 0 \cdot 2 = 105$$

$$96 + 9 + 0 = 105$$

$$\text{Determinant} = -84 - 105$$

ANSWER -189

Solve the system of equations using Cramer's rule. Check your answer using the RREF feature on your calculator.

I USED MY CALCULATOR TO EVALUATE THE DETERMINANTS

9) $6x + 2y = 10$
 $2x - y = 5$

$$D = \begin{vmatrix} 6 & 2 \\ 2 & -1 \end{vmatrix} = -10$$

$$D_x = \begin{vmatrix} 10 & 2 \\ 5 & -1 \end{vmatrix} = -20$$

$$x = \frac{D_x}{D} = \frac{-20}{-10} = 2$$

$$D_y = \begin{vmatrix} 6 & 10 \\ 2 & 5 \end{vmatrix} = 10$$

$$y = \frac{D_y}{D} = \frac{10}{-10} = -1$$

ANSWER (2, -1)

11) $4x - 3y = -2$
 $x - 5y = -9$

$$D = \begin{vmatrix} 4 & -3 \\ 1 & -5 \end{vmatrix} = -17$$

$$x = \frac{D_x}{D} = \frac{-17}{-17} = 1$$

$$D_x = \begin{vmatrix} -2 & -3 \\ -9 & -5 \end{vmatrix} = -17$$

$$y = \frac{D_y}{D} = \frac{-34}{-17} = 2$$

$$D_y = \begin{vmatrix} 4 & -2 \\ 1 & -9 \end{vmatrix} = -34$$

ANSWER (1, 2)

I used my CALCULATOR TO EVALUATE The DETERMINANTS

13) $5x + y = -7$
 $3x - 2y = -12$

$$D = \begin{vmatrix} 5 & 1 \\ 3 & -2 \end{vmatrix} = -13$$

$$x = \frac{D_x}{D} = \frac{26}{-13} = -2$$

$$D_x = \begin{vmatrix} -7 & 1 \\ -12 & -2 \end{vmatrix} = 26$$

$$y = \frac{D_y}{D} = \frac{-39}{-13} = 3$$

$$D_y = \begin{vmatrix} 5 & -7 \\ 3 & -12 \end{vmatrix} = -39$$

ANSWER (-2, 3)

15) $3x + 2y = 11$
 $2x - y = 5$

$$D = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -7$$

$$x = \frac{D_x}{D} = \frac{-21}{-7} = 3$$

$$D_x = \begin{vmatrix} 11 & 2 \\ 5 & -1 \end{vmatrix} = -21$$

$$y = \frac{D_y}{D} = \frac{-7}{-7} = 1$$

$$D_y = \begin{vmatrix} 3 & 11 \\ 2 & 5 \end{vmatrix} = -7$$

ANSWER (3, 1)

I USED MY CALCULATOR TO EVALUATE THE DETERMINANTS

17) $4x - 2y = 7$
 $2x - 5y = -3$

$$D = \begin{vmatrix} 4 & -2 \\ 2 & -5 \end{vmatrix} = -16$$

$$x = \frac{D_x}{D} = \frac{-41}{-16} = \frac{41}{16}$$

$$D_x = \begin{vmatrix} 7 & -2 \\ -3 & -5 \end{vmatrix} = -41$$

$$y = \frac{D_y}{D} = \frac{-26}{-16} = \frac{13}{8}$$

$$D_y = \begin{vmatrix} 4 & 7 \\ 2 & -3 \end{vmatrix} = -26$$

ANSWER $(\frac{41}{16}, \frac{13}{8})$

19) $5x + 2y = 22$
 $3x - 5y = 7$

$$D = \begin{vmatrix} 5 & 2 \\ 3 & -5 \end{vmatrix} = -31$$

$$x = \frac{D_x}{D} = \frac{-124}{-31} = 4$$

$$D_x = \begin{vmatrix} 22 & 2 \\ 7 & -5 \end{vmatrix} = -124$$

$$y = \frac{D_y}{D} = \frac{-31}{-31} = 1$$

$$D_y = \begin{vmatrix} 5 & 22 \\ 3 & 7 \end{vmatrix} = -31$$

ANSWER (4, 1)

I USED MY CALCULATOR TO EVALUATE THE DETERMINANTS

21)

$$-x + y + 2z = 1$$

$$2x + 3y + z = -2$$

$$5x + 4y + 2z = 4$$

$$D = \begin{vmatrix} -1 & 1 & 2 \\ 2 & 3 & 1 \\ 5 & 4 & 2 \end{vmatrix} = -15$$

$$x = \frac{D_x}{D} = \frac{-30}{-15} = 2$$

$$D_x = \begin{vmatrix} -1 & 1 & 2 \\ 2 & 3 & 1 \\ 4 & 4 & 2 \end{vmatrix} = -30$$

$$y = \frac{D_y}{D} = \frac{45}{-15} = -3$$

$$D_y = \begin{vmatrix} -1 & 1 & 2 \\ 2 & 2 & 1 \\ 5 & 4 & 2 \end{vmatrix} = 45$$

$$z = \frac{D_z}{D} = \frac{-45}{-15} = 3$$

$$D_z = \begin{vmatrix} -1 & 1 & 1 \\ 2 & 3 & 2 \\ 5 & 4 & 4 \end{vmatrix} = -45$$

ANSWER (2, -3, 3)

I used my CALCULATOR TO EVALUATE the determinants

23)

$$-5x - y + 3z = -14$$

$$-2x + 2y - 6z = 16$$

$$x + 7y + 2z = -5$$

$$D = \begin{vmatrix} -5 & -1 & 3 \\ -2 & 2 & -6 \\ 1 & 7 & 2 \end{vmatrix} = -276$$

$$X = \frac{D_x}{D} = \frac{-276}{-276} = 1$$

$$D_x = \begin{vmatrix} -14 & -1 & 3 \\ 16 & 2 & -6 \\ -5 & 7 & 2 \end{vmatrix} = -276$$

$$D_y = \begin{vmatrix} -5 & -14 & 3 \\ -2 & 16 & -6 \\ 1 & -5 & 2 \end{vmatrix} = 0$$

$$y = \frac{D_y}{D} = \frac{0}{-276} = 0$$

$$D_z = \begin{vmatrix} -5 & -1 & -14 \\ -2 & 2 & 16 \\ 1 & 7 & -5 \end{vmatrix} = 828$$

$$z = \frac{D_z}{D} = \frac{828}{-276} = -3$$

ANSWER (1, 0, -3)

I USED my CALCULATOR TO EVALUATE the DETERMINANTS

25)

$$-x + 2y - z = -17$$

$$2x - y + z = 21$$

$$3x + 2y + z = 19$$

$$D = \begin{vmatrix} -1 & 2 & -1 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = -2$$

$$x = \frac{D_x}{D} = \frac{-14}{-2} = 7$$

$$D_x = \begin{vmatrix} -17 & 2 & -1 \\ 21 & -1 & 1 \\ 19 & 2 & 1 \end{vmatrix} = -14$$

$$y = \frac{D_y}{D} = \frac{6}{-2} = -3$$

$$D_y = \begin{vmatrix} -1 & -17 & -1 \\ 2 & 21 & 1 \\ 3 & 19 & 1 \end{vmatrix} = 6$$

$$z = \frac{D_z}{D} = \frac{-8}{-2} = 4$$

$$D_z = \begin{vmatrix} -1 & 2 & -17 \\ 2 & -1 & 21 \\ 3 & 2 & 19 \end{vmatrix} = -8$$

ANSWER (7, -3, 4)

I USED MY CALCULATOR TO EVALUATE the DETERMINANTS

27)

$$4x + y + z = 9$$

$$3x - 2y + z = 4$$

$$5x - 4y + z = 6$$

$$D = \begin{vmatrix} 4 & 1 & 1 \\ 3 & -2 & 1 \\ 5 & -4 & 1 \end{vmatrix} = 0$$

$$x = \frac{D_x}{D} = \frac{16}{8} = 2$$

$$D_x = \begin{vmatrix} 9 & 1 & 1 \\ 4 & -2 & 1 \\ 6 & -4 & 1 \end{vmatrix} = 16$$

$$y = \frac{D_y}{D} = \frac{0}{0} = 0$$

$$D_y = \begin{vmatrix} 4 & 9 & 1 \\ 3 & 4 & 1 \\ 5 & 6 & 1 \end{vmatrix} = 0$$

$$z = \frac{D_z}{D} = \frac{0}{0} = 0$$

$$D_z = \begin{vmatrix} 4 & 1 & 9 \\ 3 & -2 & 4 \\ 5 & -4 & 6 \end{vmatrix} = 0$$

ANSWER (2, 1, 0)